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THEORY OF NUCLEAR BREAKUP

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FOREWORD

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THEORY OF NUCLEAR BREAKUP

The possible state of a nucleus during the instant of its
disintegration into two approximately equal nuclei is considered.
Calculation of the energy of two contiguous ellipsoids of revolution
contradicts the predictions of Ya. I. Frenkel' in favor of the exis-
tence of essentially non-spherical nuclei. The order of magnitude
obtained for the value of the energy of the ellipsoids makes it
possible to describe satisfactorily the observed formation of several
fast neutrons during each act of breakup,   as the evaporation of
these neutrons by the fragments excited during the fission process.

Bohr's theory describes the nucleus as a liquid drop with uni-

form charge density, from which electrostatic energy flows out. The short-range attraction forces of the nuclear particles causes a definite heat of their evaporation, and also surface tension of the drop.

In 1939, the most important success of the theory was the description of the fission of heavy nuclei under the influence of neutron bombardment into two approximately equal fragments, accompanied by the release of tremendous energies (100 -- 200 Mev) even for radioactive processes, and the formation of several neutrons ("neutron dust") during the breakup. This was discovered by Meitner and Frisch [1]. This last feature is of particular interest, since it opens up a principal possibility of a chain breakup of macroscopic quantities of uranium [2]. Fission theory, which has been developed in detail by three physicists -- Bohr (Denmark), Wheeler (U.S.A.) and Frenkel' (USSR) [3], there is provision for the stability of the spherical form of a uniformly-charged drop of an incompressible liquid, having definite surface tension.

It is shown that the spherical shape becomes unstable under small deformations when the ratio of the electrostatic energy E to the surface energy O becomes

$$E/O \geqslant 2. \tag{1}$$

Furthermore, all three mentioned authors consider two contiguous spherical nuclei, obtained by fission of the initial nuclei. It is

easy to determine that the energy of two contiguous spheres of half

the volume is equal to or less than the energy of the initial sphere,

if for the latter

$$E/O \geqslant 2.42. \tag{2}$$

Let us introduce a certain parameter a, which describes the

course of the fission, and for convenience in graphical representation

we shall choose a such that for   an initial nucleus of spherical

shape a = 0. * During the instant when the two produced nuclei are

contiguous in a point (which is an essential stage in the fission),

a = 1; finally, when the produced nuclei have moved an infinite dis-

tance apart, a = 2.

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* The figure 2.17 given by Frenkel' is the result of an

arithmetic error, since the corresponding formula in his article

$$\Delta W = E\left(1 - \frac{2^{1/3}}{2} - \frac{5 \cdot 2^{1/3}}{24}\right) - u\left(2^{1/3} - 1\right)$$

was written incorrectly. See also a plot of the function f* (x) (the

cited paper by Bohr and Wheeler, Fig. 4), f*(x) = 0 when x = 1.2,

where x = E/20 is the Bohr parameter.

For the value

$$2 < E/O < 2.42 \tag{3}$$

the above calculations lead to the variation of the energy during the

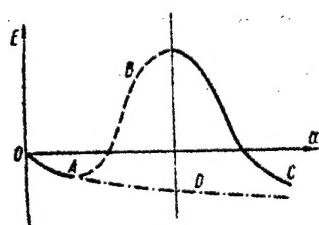course of the breakup as shown in Fig. 1 by means of the solid line.
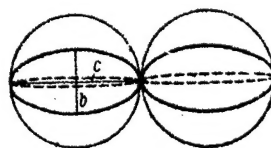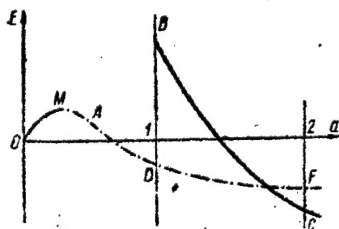
Рис. 1.
Fig. 1.

Рис. 2.
Fig. 2

Рис. 3.
Fig. 3.

The left segment OA is the result of the calculation of the small deformations of the drop xxx at $E/O > 2$; the right portion BC is the result of the calculation of the energy of two small spheres as functions of the distance. From the comparison of the two solid lines given in Fig. 1, two essentially different conclusions can be drawn.

1. By joining the solid segments with a smooth curve (dashed curve, total curve OABC) we of necessity obtain an energy minimum at $a < 1$. Physically this means that the heavy nuclei have a stable form different from spherical (Frenkel').

2. Another possibility -- disintegration through a form which differs from the two equal contiguous spheres -- "tidal perturbed form" (Bohr and Wheeler). If a sufficiently small energy corresponds to this form (point D, Fig. 1), the reasons for the ideas of stable non-spherical shapes disappear.

The calculations which are the subject of the present article pertain precisely to the energy at the instant of the disintegration, i.e., to finding the ordinates of the point D, $a = 1$.

It is easy to find that for a given charge and volume, pair-like shapes have the minimal energy at the instant of contact.

However, even calculation for ellipsoids of revolution which are prolate along the line of centers (Fig. 2) gave sufficiently definite qualitative results.

A section through the ellipsoids is shown in Fig. 2 by means of a solid line. The length of the major semi-axis is denoted by c, and that of the minor semi-axis by b. At a given ratio c/b, each of the semi-axes can be readily obtained from the condition of conservation of volume

$$2cb^2 = r_0^3, \qquad (4)$$

where r_0 is the radius of the initial nucleus.

We obtain the surface energy from the well known formula

$$O = 2\pi ab \left(\frac{b}{c} + \frac{1}{s} \arcsin s \right), \qquad (5)$$

where $\qquad s = \sqrt{1 - b^2/c^2}.$

The electrostatic energy of an individual ellipsoid is determined from the formula

$$E = \frac{3}{10} \frac{(e/2)^2}{cs} \ln \frac{(1+s)}{(1-s)}. \qquad (6)$$

It is somewhat more complicated to determine the mutual energy of the ellipsoids. An exact analytic calculation for the mutual energy of two ellipsoids, based on the method proposed by Laguerre [4] for the particular case of two homogeneous prolate ellipsoids of revolution, yields the formula

$$E_{1,2} = \frac{9}{16} \frac{(e/2)^2}{\pi} \int\limits_{-1}^{+1} \int\limits_{-1}^{+1} \int\limits_{0}^{\pi} \frac{dt\, dt_0\, d\varphi\, (1-t^2)(1-t_0^2)}{2c - t \sqrt{c^2 - b^2} - t_0 \sqrt{c^2 - b^2}}. \qquad (7)$$

The calculation of this formula has led to a very cumbersome expression, containing more than 100 terms. A calculation carried out for c/b = 2 gave results which coincide within 3% with the corresponding data of the approximate method proposed below, which was used

to carry out all the calculations.

If it is considered in addition that the mutual energy amounts to only part of the total energy, then the possible error in the calculation of the total energy will be on the order of a fraction of the percent. For a specified charge (e/2) of each of the ellipsoids and a distance 2c between centers, the electrostatic energy of interaction is obtained in an elementary manner from two limiting cases:

When b = c, spheres, the thin lines of Fig. 2:

$$E_{1,2} = \frac{1}{2c}\left(\frac{e}{2}\right)^2;$$ (8)

For b → 0 "rods," ** dotted line, Fig. 2:

$$E_{1,2} = \frac{1}{1.74c}\left(\frac{e}{2}\right)^2.$$ (9)

** On a rod obtained by a transition to the limit from an ellipsoid, the charge density is distributed not uniformly, but in accordance with the law $\rho \sim (c/2)^2 - z^2$, where z is the distance from the center of the rod.

In the region of interest to us

$$0 < b < c$$ (10)

we interpolate in accordance with the formula

$$E_{1,2} = \frac{1}{\sqrt{3.04c^2 + 0.96b^2}}\left(\frac{e}{2}\right)^2.$$ (11)

The form of formula (11) represents simplest ideas concerning

the dependence of the energy E_{12} on b, and the coefficients in (11) are chosen such as to satisfy the two limiting expressions (8) and (9).

The results of the calculations are summarized in Table 1. For different ratios E/O of the initial nucleus, we give the energies referred to the surface energy of the initial nucleus: the first column contains the energy of the initial nucleus, and the following columns the energies of the two contiguous ellipsoids obtained from the initial nucleus, for different values of c/b = 1, 2, 3, 4, and 5.

As can be seen from Table 1, the minimum energy is reached and the investigated interval of E/O when

$$3 < c/b < 4. \tag{12}$$

Table 1

1) E/O of the initial nucleus, 2) (E + O)/O of the initial nucleus

E/O исходного ядра ①	$\frac{E + O}{O}$ исх. ядра ②	1	2	3	4	5
2.60	3.60	3.581	3.414	3.318	3.312	3.493
2.40	3.40	3.402	3.254	3.118	3.180	3.293
2.20	3.20	3.231	3.095	3.038	3.050	3.05
2.00	3.00	3.047	2.939	2.898	2.918	2.929
1.80	2.80	2.869	2.779	2.755	2.789	2.807
1.70	2.70	2.778	2.70	2.686	2.723	2.747
1.65	2.65	2.734	2.659	2.650	2.69	2.718
1.60	2.55	2.689	2.620	2.614	2.658	2.687
1.40	2.40	2.511	2.462	2.473	2.527	2.565

This energy is less than the energy of the initial nucleus when E/O > 1.65.

The nucleus for which relation (3) holds cannot be in the form of a sphere. Nor can it disintegrate via contiguous spheres. Our calculation shows, however, that the disintegration via two prolate contiguous ellipsoids is not forbidden.

For the interval

$$1.65 < E/O < 2 \qquad (13)$$

our data on the variation of the energy during the breakup process are shown in Fig. 2, where all the symbols etc. are taken from Fig. 1.

The segment OMA (where M is the maximum energy) is taken from Bohr. The position of the point D, corresponding to two ellipsoids, is taken from our own data. Since in the indicated interval (13) the point D is located   below B, there are no grounds for assuming the presence of any additional minima in the interval. Confining ourselves to an analysis of spheres (segment BC), we would arrive at entirely different conclusions, which have no connection with reality.

Considering instead of ellipsoids asymmetrical pair-shaped forms, we would undoubtedly reduce the lower limit $_\wedge$(13).
of

Finally, comparing the energy at the point D with the energy of the point M, but not at the point O (Fig. 3), we should obtain $E_D <$ E_M for all values of E/O.

It is curious that after moving an infinite distance apart, the

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energy of the two of ellipsoids is naturally greater than the energy of two remote spheres (see placement of points F and C for a = 2 in Figs. 1 and 3).

The energy difference reaches 3.5 E/O ∿ 1.8 = c/b approximately 0.078 O (O is the surface energy of the initial nucleus), i.e., approximately 42 Mev for each produced nucleus. The excitation energy of the nucleus will be converted primarily into evaporation of neutrons. With a relatively low binding energy of the neutrons in a nuclei of fragments having an anomalous ratio of charge to mass, one can thus explain the emission of a considerable number of neutrons per fission event, and also the frequently observed very large neutron energies [5].

Actually, even in the case when the fission proceeds via two contiguous ellipsoids, their shape changes upon further elongation. The calculations of part of the energy of deformation of the fragments, which is converted into kinetic energy, and of that part which can be used in the form of excitation energy for evaporation of neutrons, is a problem in the dynamics of a nuclear liquid, a field which has not yet been fully developed.

The purpose of our elementary calculations consists of obtaining an indication of the order of magnitude of the possible excitation energy. In any case, the evaporation of neutrons by excited fragments appears to us more likely than the mechanism proposed by Bohr and

Wheeler. Bohr and Wheeler note that usually several small drops are placed at the point of scission when   one drop is divided into two, and identify the neutrons   precisely with these small drops.

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